Required Problems

1. Using truth tables, prove both of DeMorgan's Laws for logical connectives.
   (a) $\neg(P \wedge Q)$ is logically equivalent to $\neg P \vee \neg Q$
   (b) $\neg(P \vee Q)$ is logically equivalent to $\neg P \wedge \neg Q$

2. Let x and y be integers. Prove that if x and y are even, then $x + y$ is even.

3. Rewrite each of the following sentences to be symbolic sentences using logical connectives and quantifiers. If a quantifier's universe is included in the English sentence, be sure to include it in the symbolic sentence.
   (a) If $x = 1$ or $x = -1$, then $|x| = 1$.
   (b) B is invertible is a necessary and sufficient condition for $|B| \neq 0$.
   (c) $6 \geq n - 3$ only if $n > 8$ or $n = 9$.
   (d) Every nonzero real number is positive or negative.
   (e) S is compact iff S is closed and bounded.
   (f) Every integer is greater than some integer.

4. Let A and B be sets. Prove that $A \subseteq B$ if and only if $A - B = \emptyset$.

5. In class, I defined the uniqueness existential quantifier so that $\exists!$ means "there exists a unique". However, it can actually be defined using the symbols we already had, $\wedge, \forall, \exists$, etc. Write a symbolic sentence that is equivalent to $\exists! x \ni A(x)$ without using !.

6. Let $A = \{1, 3, 5, 7, 9\}$, $B = \{0, 2, 4, 6, 8\}$, $C = \{1, 2, 4, 5, 7, 8\}$, and $D = \{1, 2, 3, 5, 6, 7, 8, 9, 10\}$. Find the following:
   (a) $A \cup B$
   (b) $A - B$
   (c) $(A \cap C) \cap D$
   (d) $A \cup (C \cap D)$

Optional Problems

7. Let x and y be integers. Prove the following propositions:
   (a) If x and y are even, then xy is even.
   (b) If x and y are odd, then $x + y$ is even.
   (c) If X is even and y is odd, then $x + y$ is odd.

8. Let x be an integer. Write a proof by contraposition to show that if x is even, then $x + 1$ is odd.

9. Suppose a and b are positive integers. Write a proof by contradiction to show that if ab is odd, the both a and b are odd.

10. Let A, B, C, and D be sets. Prove that if $C \subseteq A$ and $D \subseteq B$ and A and B are disjoint, then C and D are disjoint.

11. Find the contrapositive and converse of each of the following statements, writing as "if-then" statements:

    (a) "If squares have four sides, then triangles have four sides."

    (b) "A sequence a is bounded whenever a is convergent."

    (c) "The differentiability of a function f is sufficient for f to be continuous."