Required Problems

1. Consider the sequence $\{x_n\}_{n=1}^{\infty}$ such that
$$x_n = \frac{n+1}{n}$$
To what does this sequence converge? Prove that this sequence converges to that limit.

2. Let S and T be convex sets. Prove that the intersection of S and T is also a convex set.

3. The set $S^{n-1} = \{\mathbf{x} \mid \sum_{i=1}^{n} x_i = 1 \ \wedge \ x_i \geq 0 \ \forall i = 1, \ldots, n\}$ is the $(n-1)$-dimensional unit simplex.
   (a) Describe in words the set S^{n-1} for $n = 3$
   (b) Prove that S^{n-1} is a convex set.
   (c) Prove that S^{n-1} is a compact set.

4. Let D be a convex subset of $\mathbb{R}^n$ and $f : D \to \mathbb{R}$. For the following two statements, if it is true provide a proof. If it is false, provide a counterexample.
   (a) f is strictly concave $\implies$ f is strictly quasiconcave
   (b) f is strictly quasiconcave $\implies$ f is strictly concave

Additional Practice Problems (I will provide solutions for these but not feedback)

5. Give a relation r from $A = \{5, 6, 7\}$ to $B = \{3, 4, 5\}$ such that
   (a) r is not a function
   (b) r is a function from A to B with the range $\mathcal{R}(r) = B$
   (c) r is a function from A to B with the range $\mathcal{R}(r) \neq B$

6. Identify the domain and range of each of the following mappings:
   (a) $\left\{(x, y) \in \mathbb{R}^2 \middle| y = \frac{1}{x+1}\right\}$
   (b) $\left\{(x, y) \in \mathbb{N} \times \mathbb{N} \middle| y = x + 5\right\}$
   (c) $\left\{(x, y) \in \mathbb{Z} \times \mathbb{Z} \middle| y = \frac{x^2-4}{x-2}\right\}$

7. For each of the following sequences, list the first three terms:
   (a) $a_n = \frac{n+1}{2n+3}$
   (b) $b_n = \frac{1}{n!}$
   (c) $c_n = 1 - 2^{-n}$

8. Prove that if $x_n \to L$ and $y_n \to M$, then $x_n + y_n \to L + M$.

9. Prove that if $a_n \to a$ and $a_n \leq b$ for all n, then $a \leq b$.

10. Consider the following intervals in $\mathbb{R}$. For each, determine if it is closed. If so, give a proof:
    (a) $(-\infty, b]$
    (b) $(a, b]$
    (c) $[a, \infty)$
    (d) $[a, b)$

11. Consider the following sets. If the set is bounded, provide an M and a $\mathbf{x}$ such that $B_M(\mathbf{x})$ contains the set.
    (a) $A = \{x | x \in \mathbb{R} \wedge x^2 \leq 10\}$
    (b) $B = \left\{x | x \in \mathbb{R} \wedge x + \frac{1}{x} < 5\right\}$
    (c) $C = \{(x, y) | (x, y) \in \mathbb{R}^2_+ \wedge xy < 1\}$
    (d) $D = \{(x, y) | (x, y) \in \mathbb{R} \wedge |x| + |y| \leq 10\}$

12. Prove that the following functions are continuous using epsilon-delta proofs.
    (a) $f(x) = x + 3$
    (b) $g(x) = x^2$
    (c) $h(x) = |x|$