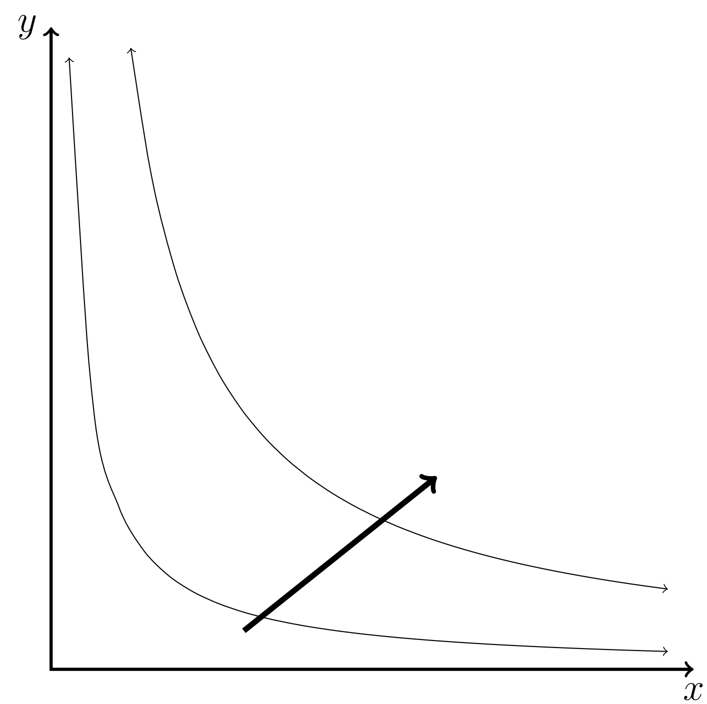
- Convex set: $t\mathbf{x}_0 + (1-t)\mathbf{x}_1 \in A \quad \forall\ \mathbf{x}_0, \mathbf{x}_1 \in A, t \in [0,1]$
- Superior set for $\mathbf{x}_0$: $\{\mathbf{x} \mid \mathbf{x} \in D \wedge f(\mathbf{x}) \geq f(\mathbf{x}_0)\}$
- Inferior set for $\mathbf{x}_0$: $\{\mathbf{x} \mid \mathbf{x} \in D \wedge f(\mathbf{x}) \leq f(\mathbf{x}_0)\}$
- Quasiconcave function: $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \geq \min\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \;\; \forall t \in [0,1]$
- Strictly quasiconcave function: $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \;\; \forall \mathbf{x}_0 \neq \mathbf{x}_1,\; t \in (0,1)$
- Quasiconvex function: $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \leq \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \;\; \forall t \in [0,1]$
- Strictly quasiconvex function: $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\{f(\mathbf{x}_0), f(\mathbf{x}_0)\} \;\; \forall \mathbf{x}_0 \neq \mathbf{x}_1,\; t \in (0,1)$
- Concave shape: A line connecting two points is underneath
- Strictly concave shape: A line connecting two points is strictly underneath
- Convex shape: A line connecting two points is above
- Strictly convex shape: A line connecting two points is strictly above
- Homothetic function: level sets have the same slope along rays from the origin

Cobb-Douglas Utility Function: $u(x,y) = x^{\alpha}\, y^{\beta}$

1. Superior set is always convex? Yes
2. Inferior set is always convex?
3. u is a quasiconcave function? Yes
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \geq \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \;\; \forall t \in [0,1]$
4. u is a strictly quasiconcave function? Yes
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \;\forall \mathbf{x}_0 \neq \mathbf{x}_1,\; t \in (0,1)$
5. u is a quasiconvex function?
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \leq \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \;\; \forall t \in [0,1]$
6. u is a strictly quasiconvex function?
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\left\{f(\mathbf{x}_0), f(\mathbf{x}_0)\right\} \;\forall \mathbf{x}_0 \neq \mathbf{x}_1,\; t \in (0,1)$

The level sets (indfference curves) have a . . .

7. . . . concave shape?
8. . . . strictly concave shape?
9. . . . convex shape? Yes
10. . . . strictly convex shape? Yes
11. u is homothetic? Yes



If both goods are "goods", then the indifference curves are downward sloping and increasing to the upper right.

Perfect Substitutes Utility Function: $u(x, y) = ax + by$

1. Superior set is always convex? Yes

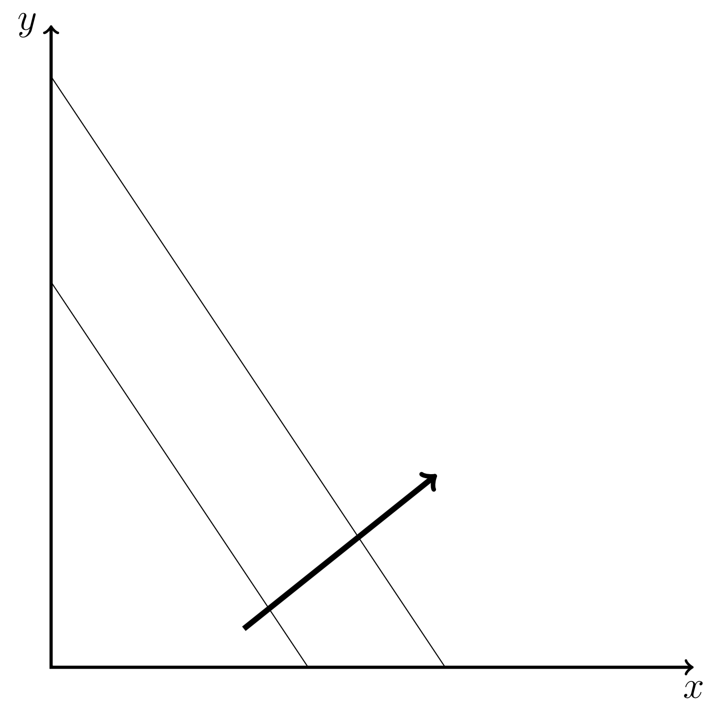
2. Inferior set is always convex? Yes

3. u is a quasiconcave function? Yes
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \geq \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \;\; \forall t \in [0,1]$

4. u is a strictly quasiconcave function?
   $f(t\mathbf{x}_0+(1-t)\mathbf{x}_1) > \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \; \forall \mathbf{x}_0 \neq \mathbf{x}_1, \; t \in (0,1)$

5. u is a quasiconvex function? Yes
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \leq \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \;\; \forall t \in [0,1]$

6. u is a strictly quasiconvex function?
   $f(t\mathbf{x}_0+(1-t)\mathbf{x}_1) < \max\left\{f(\mathbf{x}_0), f(\mathbf{x}_0)\right\} \; \forall \mathbf{x}_0 \neq \mathbf{x}_1, \; t \in (0,1)$



The level sets (indfference curves) have a ...

7. ...concave shape? Yes

8. ...strictly concave shape?

9. ...convex shape? Yes

10. ...strictly convex shape?

11. u is homothetic? Yes

Perfect Complements Utility Function: $u(x, y) = \min\{f(x), g(y)\}$

1. Superior set is always convex? Yes

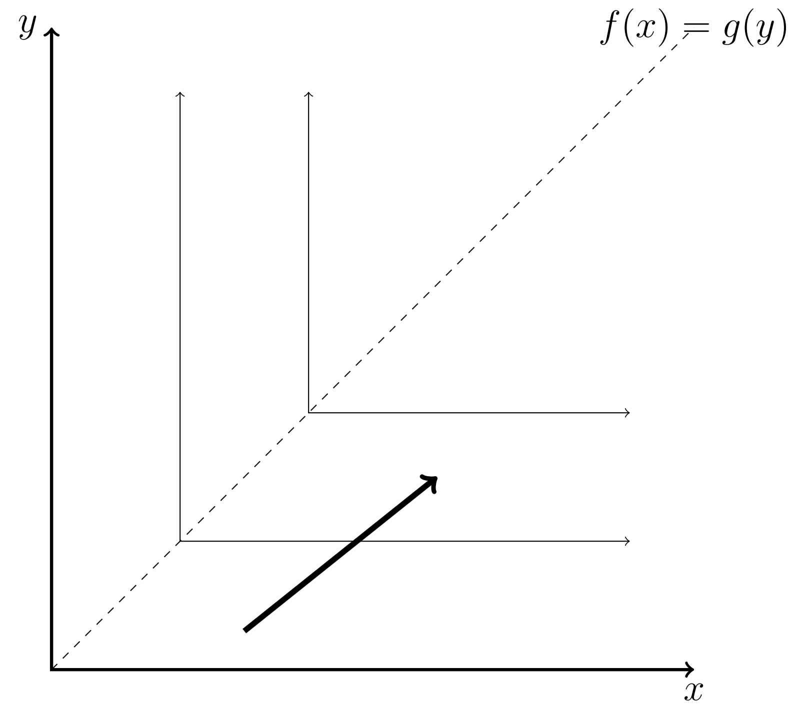
2. Inferior set is always convex?

3. u is a quasiconcave function? Yes
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \geq \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \;\; \forall t \in [0, 1]$

4. u is a strictly quasiconcave function?
   $f(t\mathbf{x}_0+(1-t)\mathbf{x}_1) > \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \;\forall \mathbf{x}_0 \neq \mathbf{x}_1,\; t \in (0, 1)$

5. u is a quasiconvex function?
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \leq \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \;\; \forall t \in [0, 1]$

6. u is a strictly quasiconvex function?
   $f(t\mathbf{x}_0+(1-t)\mathbf{x}_1) < \max\left\{f(\mathbf{x}_0), f(\mathbf{x}_0)\right\} \;\forall \mathbf{x}_0 \neq \mathbf{x}_1,\; t \in (0, 1)$



The level sets (indfference curves) have a ...

7. ...concave shape?

8. ...strictly concave shape?

9. ...convex shape? Yes

10. ...strictly convex shape?

11. u is homothetic? Only in special cases

Prefers Extremes Utility Function: $u(x, y) = x^2 + y^2$

1. Superior set is always convex?

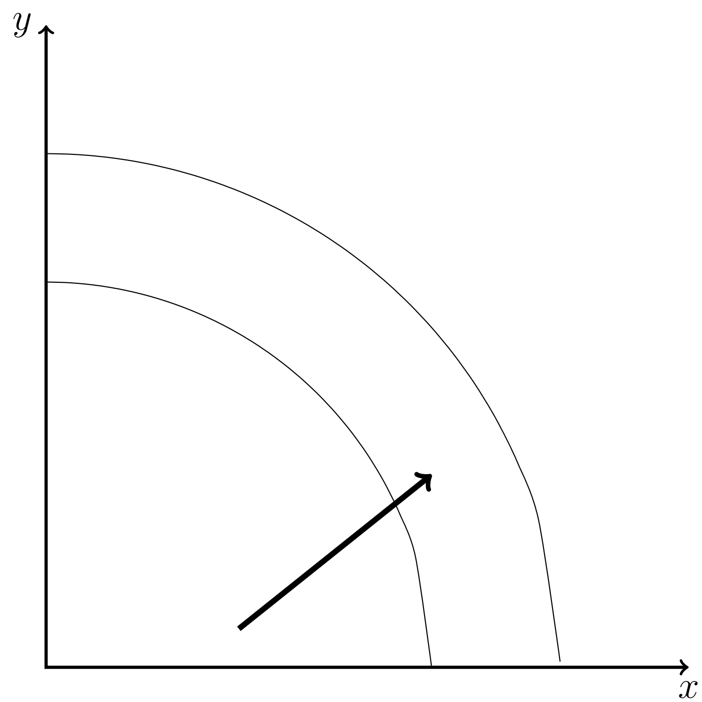
2. Inferior set is always convex? Yes

3. u is a quasiconcave function?
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \geq \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \;\; \forall t \in [0, 1]$

4. u is a strictly quasiconcave function?
   $f(t\mathbf{x}_0+(1-t)\mathbf{x}_1) > \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \; \forall \mathbf{x}_0 \neq \mathbf{x}_1, \; t \in (0, 1)$

5. u is a quasiconvex function? Yes
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \leq \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \;\; \forall t \in [0, 1]$

6. u is a strictly quasiconvex function? Yes
   $f(t\mathbf{x}_0+(1-t)\mathbf{x}_1) < \max\left\{f(\mathbf{x}_0), f(\mathbf{x}_0)\right\} \; \forall \mathbf{x}_0 \neq \mathbf{x}_1, \; t \in (0, 1)$



The level sets (indfference curves) have a ...

7. ...concave shape? Yes

8. ...strictly concave shape? This one is

9. ...convex shape?

10. ...strictly convex shape?

11. u is homothetic? This one is

Quasilinear Utility Function: $u(x, y) = x + \ln(y)$

1. Superior set is always convex? Yes

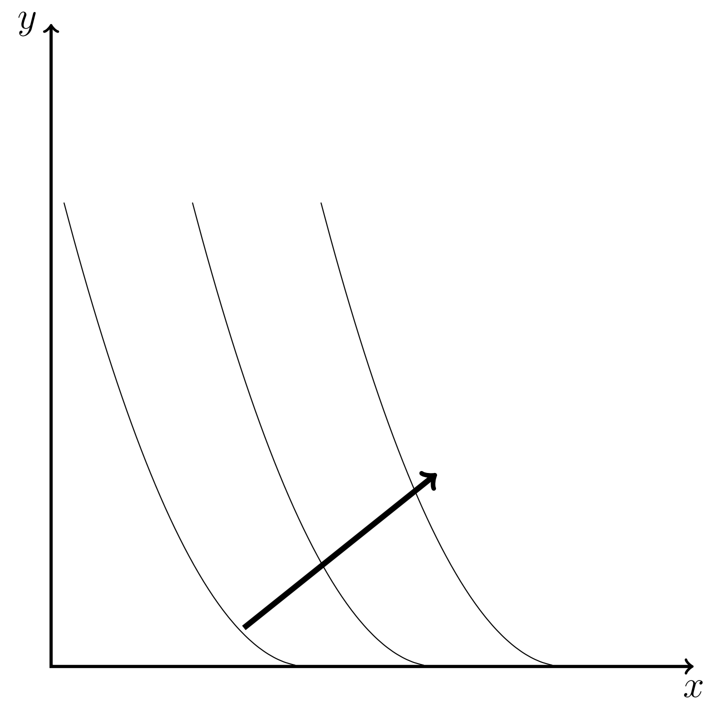
2. Inferior set is always convex?

3. u is a quasiconcave function? Yes
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \geq \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \;\; \forall t \in [0,1]$

4. u is a strictly quasiconcave function? Yes
   $f(t\mathbf{x}_0+(1-t)\mathbf{x}_1) > \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \; \forall \mathbf{x}_0 \neq \mathbf{x}_1, \; t \in (0,1)$

5. u is a quasiconvex function?
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \leq \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \;\; \forall t \in [0,1]$

6. u is a strictly quasiconvex function?
   $f(t\mathbf{x}_0+(1-t)\mathbf{x}_1) < \max\left\{f(\mathbf{x}_0), f(\mathbf{x}_0)\right\} \; \forall \mathbf{x}_0 \neq \mathbf{x}_1, \; t \in (0,1)$



The level sets (indfference curves) have a . . .

7. . . . concave shape?

8. . . . strictly concave shape?

9. . . . convex shape? Yes

10. . . . strictly convex shape? Yes

11. u is homothetic? No, but IDCs are parallel along x-axis

MRS Test

$$MRS(x,y) = \frac{\partial u/\partial x}{\partial u/\partial y}$$

What happens to the MRS as $x \uparrow$?

$$\frac{\partial MRS}{\partial x} =$$

What happens to the MRS as $y \downarrow$?

$$\frac{-\partial MRS}{\partial y} =$$

- $u(x,y) = x^{\alpha}\ y^{\beta}$
  - $MRS = \frac{\alpha y}{\beta x}$
  - $x \uparrow: \downarrow \qquad y \downarrow: \downarrow$
  - Strictly diminishing MRS
- $u(x,y) = ax + by$
  - $MRS = \frac{a}{b}$
  - $x \uparrow: \rightarrow \qquad y \downarrow: \rightarrow$
  - Constant MRS
- $u(x,y) = \min\{f(x), g(y)\}$
  - Perfect complements (MRS is 0 or ∞)
- $u(x,y) = x^2 + y^2$
  - $MRS = \frac{x}{y}$
  - $x \uparrow: \uparrow \qquad y \downarrow: \uparrow$
  - Increasing MRS (also OK if one is constant)
- $u(x,y) = x + \ln(y)$
  - $MRS = \frac{y}{1/y} = y$
  - $x \uparrow: \downarrow \qquad y \downarrow: \rightarrow$
  - Diminishing but not strictly MRS
  - Worried about IDCs that cross the axes (watch out for negative bundles)