- Convex set: $t\mathbf{x}_0 + (1-t)\mathbf{x}_1 \in A \quad \forall\ \mathbf{x}_0, \mathbf{x}_1 \in A, t \in [0,1]$

- Superior set for $\mathbf{x}_0$: $\left\{\mathbf{x} \mid \mathbf{x} \in D \wedge f(\mathbf{x}) \geq f(\mathbf{x}_0)\right\}$

- Inferior set for $\mathbf{x}_0$: $\left\{\mathbf{x} \mid \mathbf{x} \in D \wedge f(\mathbf{x}) \leq f(\mathbf{x}_0)\right\}$

- Quasiconcave function: $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \geq \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \quad \forall t \in [0,1]$

- Strictly quasiconcave function: $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \quad \forall \mathbf{x}_0 \neq \mathbf{x}_1,\ t \in (0,1)$

- Quasiconvex function: $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \leq \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \quad \forall t \in [0,1]$

- Strictly quasiconvex function: $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\left\{f(\mathbf{x}_0), f(\mathbf{x}_0)\right\} \quad \forall \mathbf{x}_0 \neq \mathbf{x}_1,\ t \in (0,1)$

- Concave shape: A line connecting two points is underneath

- Strictly concave shape: A line connecting two points is strictly underneath

- Convex shape: A line connecting two points is above

- Strictly convex shape: A line connecting two points is strictly above

- Homothetic function: level sets have the same slope along rays from the origin

# Cobb-Douglas Utility Function: $u(x,y) = x^\alpha\, y^\beta$

1. Superior set is always convex?

2. Inferior set is always convex?

3. $u$ is a quasiconcave function?
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \geq \min\{f(\mathbf{x}_0), f(\mathbf{x}_1)\}\ \ \forall t \in [0,1]$

4. $u$ is a strictly quasiconcave function?
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min\{f(\mathbf{x}_0), f(\mathbf{x}_1)\}\ \ \forall \mathbf{x}_0 \neq \mathbf{x}_1,\ t \in (0,1)$

5. $u$ is a quasiconvex function?
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \leq \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\}\ \ \forall t \in [0,1]$

6. $u$ is a strictly quasiconvex function?
   $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\{f(\mathbf{x}_0), f(\mathbf{x}_0)\}\ \ \forall \mathbf{x}_0 \neq \mathbf{x}_1,\ t \in (0,1)$

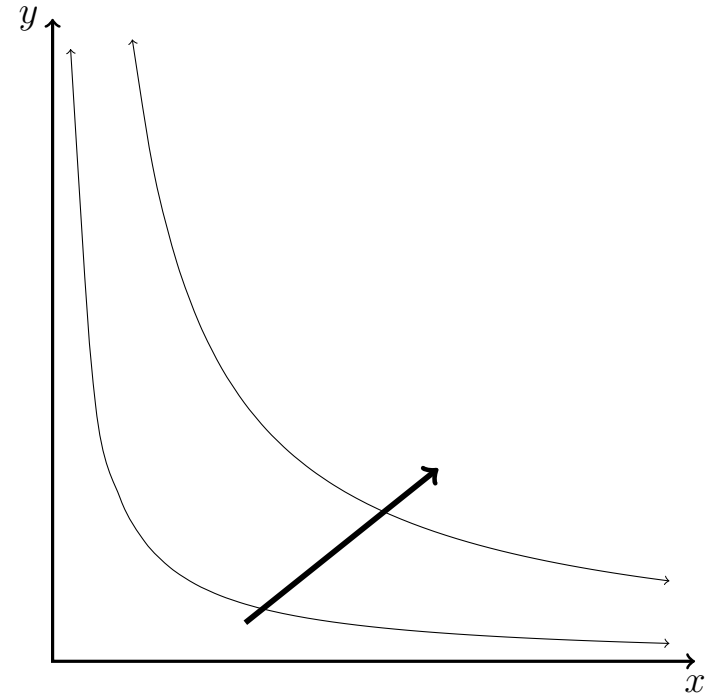The level sets (indfference curves) have a . . .

7. . . . concave shape?

8. . . . strictly concave shape?

9. . . . convex shape?

10. . . . strictly convex shape?

11. $u$ is homothetic?



If both goods are "goods", then the indifference curves are downward sloping and increasing to the upper right.

# Perfect Substitutes Utility Function: $u(x, y) = ax + by$

1. Superior set is always convex?

2. Inferior set is always convex?

3. $u$ is a quasiconcave function?
   $$f(t\mathbf{x}_0 + (1 - t)\mathbf{x}_1) \geq \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \quad \forall t \in [0, 1]$$

4. $u$ is a strictly quasiconcave function?
   $$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \quad \forall \mathbf{x}_0 \neq \mathbf{x}_1,\ t \in (0, 1)$$

5. $u$ is a quasiconvex function?
   $$f(t\mathbf{x}_0 + (1 - t)\mathbf{x}_1) \leq \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \quad \forall t \in [0, 1]$$

6. $u$ is a strictly quasiconvex function?
   $$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\left\{f(\mathbf{x}_0), f(\mathbf{x}_0)\right\} \quad \forall \mathbf{x}_0 \neq \mathbf{x}_1,\ t \in (0, 1)$$

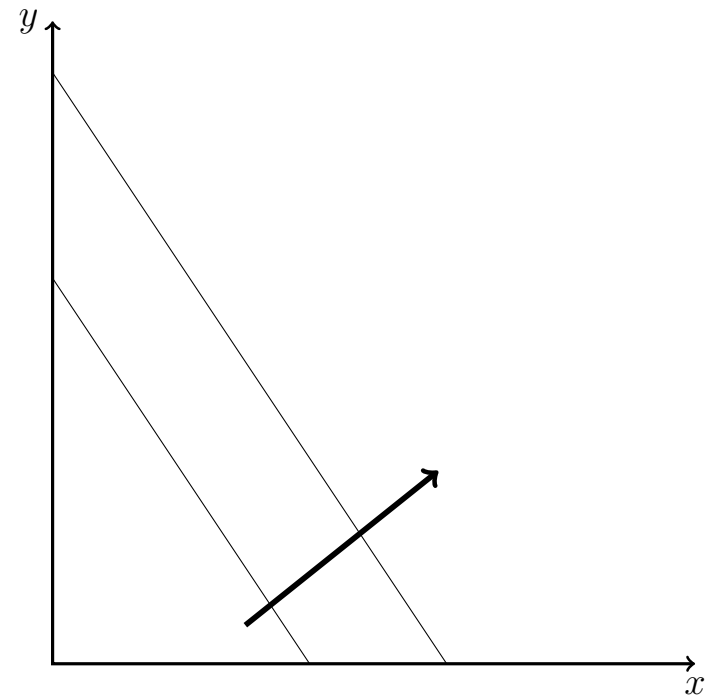The level sets (indfference curves) have a . . .

7. . . . concave shape?

8. . . . strictly concave shape?

9. . . . convex shape?

10. . . . strictly convex shape?

11. $u$ is homothetic?

# Perfect Complements Utility Function: $u(x,y) = \min\{f(x), g(y)\}$

1. Superior set is always convex?

2. Inferior set is always convex?

3. $u$ is a quasiconcave function?
   $$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \geq \min\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \quad \forall t \in [0,1]$$

4. $u$ is a strictly quasiconcave function?
   $$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \quad \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0,1)$$

5. $u$ is a quasiconvex function?
   $$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \leq \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \quad \forall t \in [0,1]$$

6. $u$ is a strictly quasiconvex function?
   $$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\{f(\mathbf{x}_0), f(\mathbf{x}_0)\} \quad \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0,1)$$

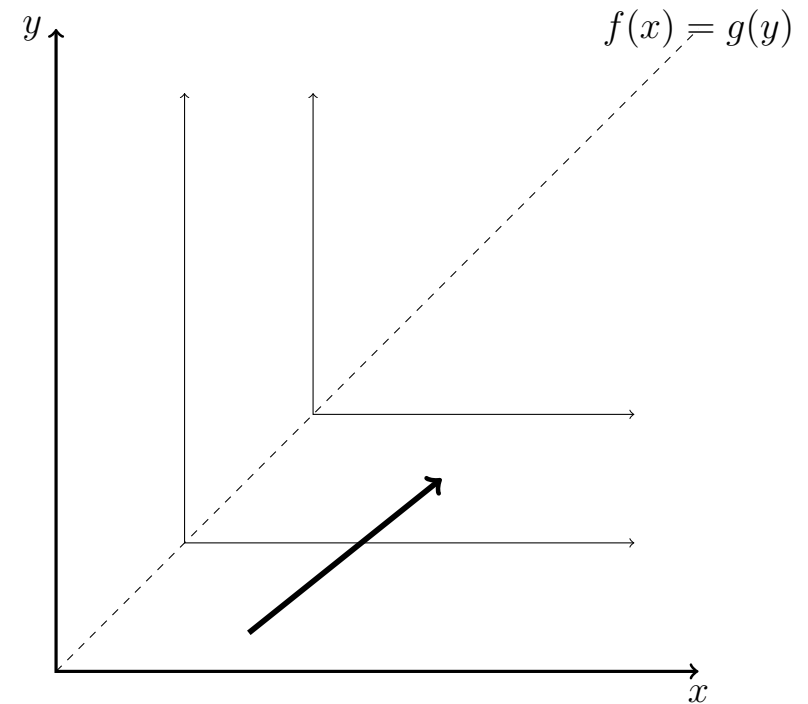The level sets (indfference curves) have a ...

7. ...concave shape?

8. ...strictly concave shape?

9. ...convex shape?

10. ...strictly convex shape?

11. $u$ is homothetic?



4

# Prefers Extremes Utility Function: $u(x, y) = x^2 + y^2$

1. Superior set is always convex?

2. Inferior set is always convex?

3. $u$ is a quasiconcave function?
   $$f(t\mathbf{x}_0 + (1 - t)\mathbf{x}_1) \geq \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \quad \forall t \in [0, 1]$$

4. $u$ is a strictly quasiconcave function?
   $$f(t\mathbf{x}_0 + (1 - t)\mathbf{x}_1) > \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \quad \forall \mathbf{x}_0 \neq \mathbf{x}_1,\ t \in (0, 1)$$

5. $u$ is a quasiconvex function?
   $$f(t\mathbf{x}_0 + (1 - t)\mathbf{x}_1) \leq \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \quad \forall t \in [0, 1]$$

6. $u$ is a strictly quasiconvex function?
   $$f(t\mathbf{x}_0 + (1 - t)\mathbf{x}_1) < \max\left\{f(\mathbf{x}_0), f(\mathbf{x}_0)\right\} \quad \forall \mathbf{x}_0 \neq \mathbf{x}_1,\ t \in (0, 1)$$
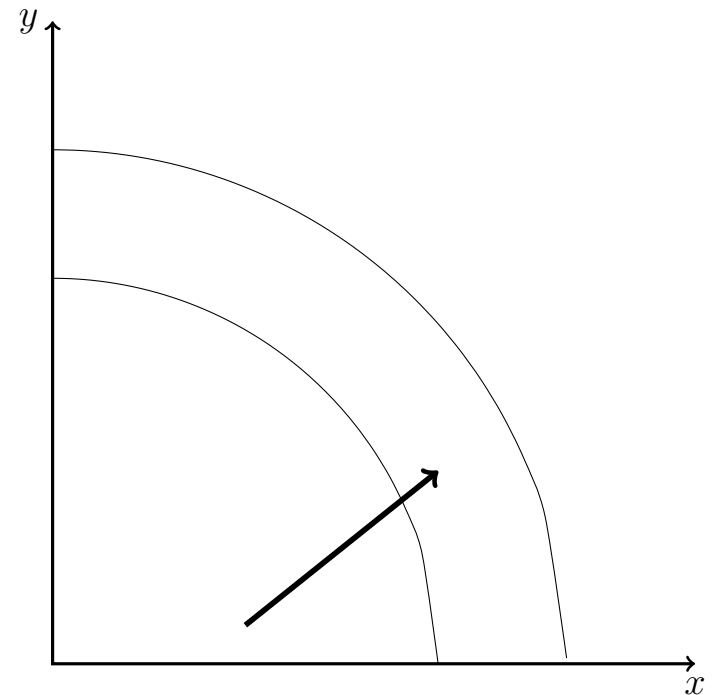
The level sets (indfference curves) have a ...

7. ...concave shape?

8. ...strictly concave shape?

9. ...convex shape?

10. ...strictly convex shape?

11. $u$ is homothetic?

# Quasilinear Utility Function: $u(x,y) = x + \ln(y)$

1. Superior set is always convex?

2. Inferior set is always convex?

3. $u$ is a quasiconcave function?
$$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \geq \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \quad \forall t \in [0,1]$$

4. $u$ is a strictly quasiconcave function?
$$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min\left\{f(\mathbf{x}_0), f(\mathbf{x}_1)\right\} \quad \forall \mathbf{x}_0 \neq \mathbf{x}_1, \; t \in (0,1)$$

5. $u$ is a quasiconvex function?
$$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \leq \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \quad \forall t \in [0,1]$$

6. $u$ is a strictly quasiconvex function?
$$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\left\{f(\mathbf{x}_0), f(\mathbf{x}_0)\right\} \quad \forall \mathbf{x}_0 \neq \mathbf{x}_1, \; t \in (0,1)$$
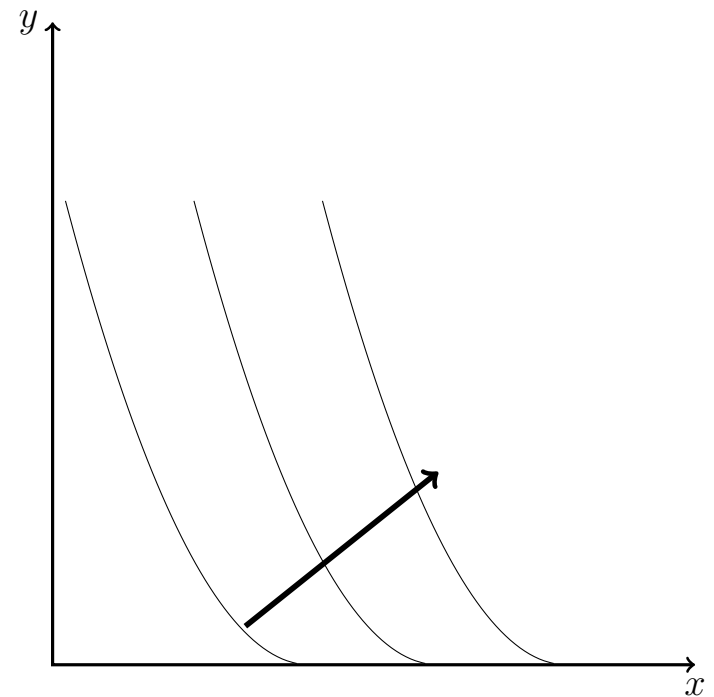
The level sets (indfference curves) have a ...

7. ...concave shape?

8. ...strictly concave shape?

9. ...convex shape?

10. ...strictly convex shape?

11. $u$ is homothetic?

# MRS Test

$$MRS(x, y) = \frac{\partial u/\partial x}{\partial u/\partial y}$$

- $u(x, y) = x^\alpha \ y^\beta$

- $u(x, y) = ax + by$

What happens to the $MRS$ as $x \uparrow$?

$$\frac{\partial MRS}{\partial x} =$$

- $u(x, y) = \min\{f(x), g(y)\}$

What happens to the $MRS$ as $y \downarrow$?

- $u(x, y) = x^2 + y^2$

$$\frac{-\partial MRS}{\partial y} =$$

- $u(x, y) = x + \ln(y)$